Exercise 14

A curve passes through the point $(0, 5)$ and has the property that the slope of the curve at every point P is twice the y-coordinate of P. What is the equation of the curve?

Solution

Write a differential equation from the property that the slope of the curve at every point is twice the y-coordinate.

$$\frac{dy}{dx} = 2y$$

Divide both sides by y.

$$\frac{1}{y}\frac{dy}{dx} = 2$$

Rewrite the left side by using the chain rule.

$$\frac{d}{dx}\ln y = 2$$

The function you have to take the derivative of to get 2 is $2x + C$, where C is any constant.

$$\ln y = 2x + C$$

Exponentiate both sides to get y.

$$e^{\ln y} = e^{2x+C}$$

$$y = e^C e^{2x}$$

Use a new constant A for e^C.

$$y(x) = Ae^{2x}$$

Use the fact that the curve passes through $(0, 5)$ to determine A.

$$5 = Ae^{2(0)} \quad \rightarrow \quad A = 5$$

Therefore,

$$y(x) = 5e^{2x}.$$

Just to check, the slope of this curve is

$$\frac{dy}{dx} = \frac{d}{dx}(5e^{2x}) = 5e^{2x} \cdot \frac{d}{dx}(2x) = 5e^{2x} \cdot (2) = 2(5e^{2x}) = 2y$$

as expected.